Data-driven Compression

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CNB G 100.9
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Lossless Compression

\( x \in A \)

size( \( x \)) is the size of the object \( x \) compressed using a lossless compressor

\[
\text{size}(x) < \text{size}(x)
\]
Lossy Compression

\( x \in A \)

\( f: A \rightarrow B \)  
Lossy Compressor

\( f(x) \)  
Compressed version of \( x \)

such that:

\[ \left\| x - f(x) \right\|_p \]  
is small

\[ \text{size}(f(x)) < \text{size}(x) \]
Data-Driven Compression, i.e., PCA

\[ x \in \mathbb{R}^n \text{ r. v.} \]
\[ x \sim N(\mu, \Sigma) \]

\[ G \text{ is an affine subspace of } \mathbb{R}^n \]
\[ \text{dim}(G) \ll n \]

\[ \text{Pr}: \mathbb{R}^n \rightarrow G \]
\[ \|x - \text{Pr}(x)\|_2 \text{ is small} \]

\[ \text{Pr}(x) \text{ is the compressed version of } x \]
\[ \text{size}(\text{Pr}(x)) = \text{dim}(G) < n = \text{size}(x) \]
Data-Driven Compression Example

\[ x \in \mathbb{R}^n \text{ r. v.} \quad \text{images representing faces} \]

\[ x \sim N(\mu, \Sigma) \]

AT&T face database
Downloadable from the course website
40 people
10 expressions each
Data-Driven Compression Example

\[ x \in \mathbb{R}^n \text{ r. v.} \text{ images representing faces} \]
\[ x \sim N(\mu, \Sigma) \]

Each face has to be treated as a vector of dimension
\[ n = \text{dimX} \times \text{dimY} \]

\text{dimX and dimY are the sizes of the image}
Data-Driven Compression Example

\( x \in \mathbb{R}^n \) r. v. \( \text{images representing faces} \)

\( x \sim N(\mu, \Sigma) \)

Note: we assume that it can be approximate as a Gaussian distribution

Compute the mean \( \mu \) and the covariance matrix \( \Sigma \) of \( x \) given the sample \( x_i \) provided by this dataset:

\[
\mu = \frac{1}{q} \sum_{i=1}^{q} x_i
\]

Using \( q \) samples

\[
\Sigma = \frac{1}{q-1} \sum_{i=1}^{q} (x_i - \mu)(x_i - \mu)^T
\]
Data-Driven Compression Example

\( \mu \in \mathbb{R}^n \)

\( \sum \in \mathbb{R}^{(n)x(n)} \)

It cannot be easily visualized
Data-Driven Compression Example

\( x \in \mathbb{R}^n \) r. v. images representing faces
\( x \sim N(\mu, \Sigma) \)

Compute \( G \) of dimension \( m \) such that it minimizes

\[
E(\| x - \text{Pr}(x) \|_2)
\]

Solution:
\( G \) is the affine space passing through \( \mu \) generated by the span of the first \( m \) eigenvectors of \( \Sigma \)
Resize the images around the 60%  // avoid memory issues

\[ L = \begin{bmatrix} (x_1 - \mu), (x_2 - \mu), \ldots, (x_q - \mu) \end{bmatrix}; \]

\[ \Sigma = \frac{1}{(q-1)} LL^T \]

\([V,D]=\text{eig}(\Sigma); \quad \text{// the columns of } V \text{ contains} \]

the eigenvectors of \( \Sigma \)
The Eigenvectors of $\Sigma$

The first 10 eigenvector of $\Sigma$ ordered by their respectively eigenvalues

G
Comprehension

% Mean( ) stores the face mean
% V stores the eigenvectors

% Generate the affine subspace G using the first space_dimension eigenvectors
affinesubspace( :, 1 : space_dimension ) = V( :, 1 : space_dimension );

% Load face
A = imread( sprintf( 's%i\%i.pgm', person_id, expression ));
A = imresize( A, IMG_SCALE, 'bicubic' );

% Project into the affine subspace G
Pr = double( A ) - Mean;
local_coords = affinesubspace' * Pr( : );

% local_coords is represented using space_dimension values

% Reproject it back to the original coordinates
RePr = reshape( (( affinesubspace * local_coords ) + Mean( : ) ), size( A, 1 ), size( A, 2 ) );

Projection of A in the local coordinates of G

\[ \text{Pr} : \mathbb{R}^n \rightarrow G \]
Example

In our case $n = 68 \times 56 = 3808$ bytes

Let $\dim(G) = \text{space\_dimension} = 100$

We are going to represent one face using only 100 floating point values (almost 400 bytes)
Example

\[ \text{dim}(G) = 50 \]

\[ \text{dim}(G) = 200 \]
Matlab interactive demo....

- Evaluation of the compression ratio...
- Evaluation of the Generalization properties...
George goes to holyday!!!

Assignment:
Find the x-position of George’s head in the above image...
Create the eigenfaces using only the first 20 people
Try to compress each part of the image using the above described compressor
Evaluate the compression error (using SSD)
The one with the lowest error is George!!!
George goes to holyday!!!
Questions???
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