Machine Learning

Central Problem of Pattern Recognition: Supervised and Unsupervised Learning

Classification

Bayesian Decision Theory

Perceptrons and SVMs

Clustering

Dimension Reduction

The Problem of Pattern Recognition

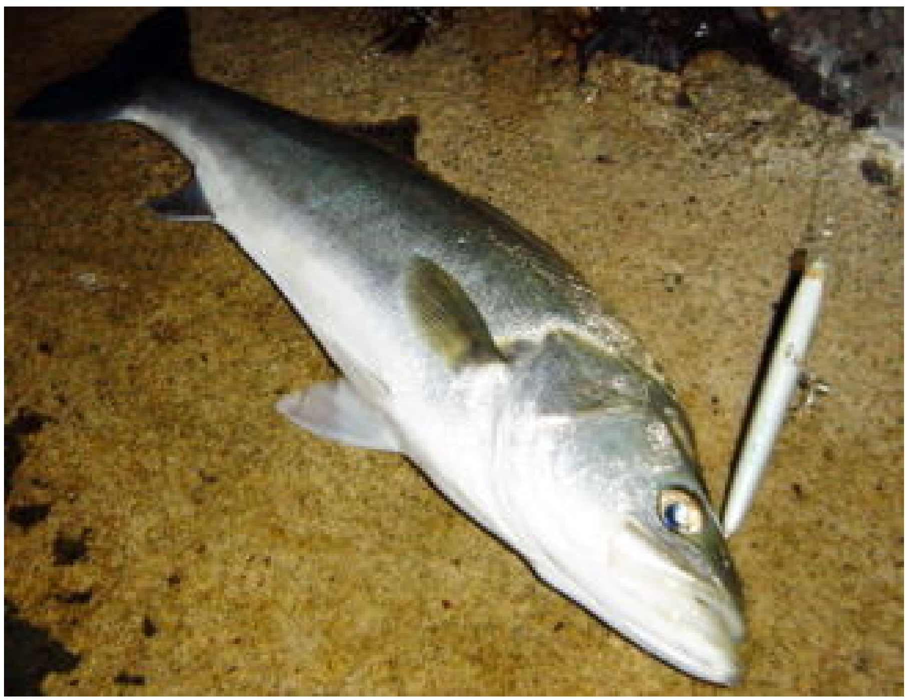
Machine Learning (as statistics) addresses a number of challenging *inference* problems in pattern recognition which span the range from statistical modeling to efficient algorithmics. *Approximative method* which yield *good performance on average* are particularly important.

- **Representation of objects**. $\Rightarrow$ Data representation
- What is a *pattern*? **Definition/modeling of structure**.
- **Optimization**: Search for prefered structures
- **Validation**: are the structures indeed in the data or are they explained by fluctuations?

Literatur

- Richard O. Duda, Peter E. Hart & David G. Stork, *Pattern Classification*. Wiley & Sons (2001)
- Trevor Hastie, Robert Tibshirani & Jerome Friedman, *The Elements of Statistical Learning: Data Mining, Inference and Prediction*. Springer Verlag (2001)
- Luc Devroye, Laslo Györfi & Gabor Lugosi, *A Probabilistic Theory of Pattern Recognition*. Springer Verlag (1996)
- Vladimir N. Vapnik, *Estimation of Dependences Based on Empirical Data*. Springer Verlag (1983); *The Nature of Statistical Learning Theory*. Springer Verlag (1995)
- Larry Wasserman, *All of Statistics*. (1st ed. 2004. Corr. 2nd printing, ISBN: 0-387-40272-1) Springer Verlag (2004)

The Classification Problem




bass

Classification as a Pattern Recognition Problem

Problem: We look for a partition of the object space $\mathcal{O}$ (fish in the previous example) which corresponds to classification examples.

Distinguish conceptually between “objects” $o \in \mathcal{O}$ and “data” $x \in \mathcal{X}$!

Data: pairs of **feature vectors** and **class labels**

$$\mathcal{Z} = \{(x_i, y_i) : 1 \le i \le n, x_i \in \mathbb{R}^d, y_i \in \{1, \ldots, k\}\}$$

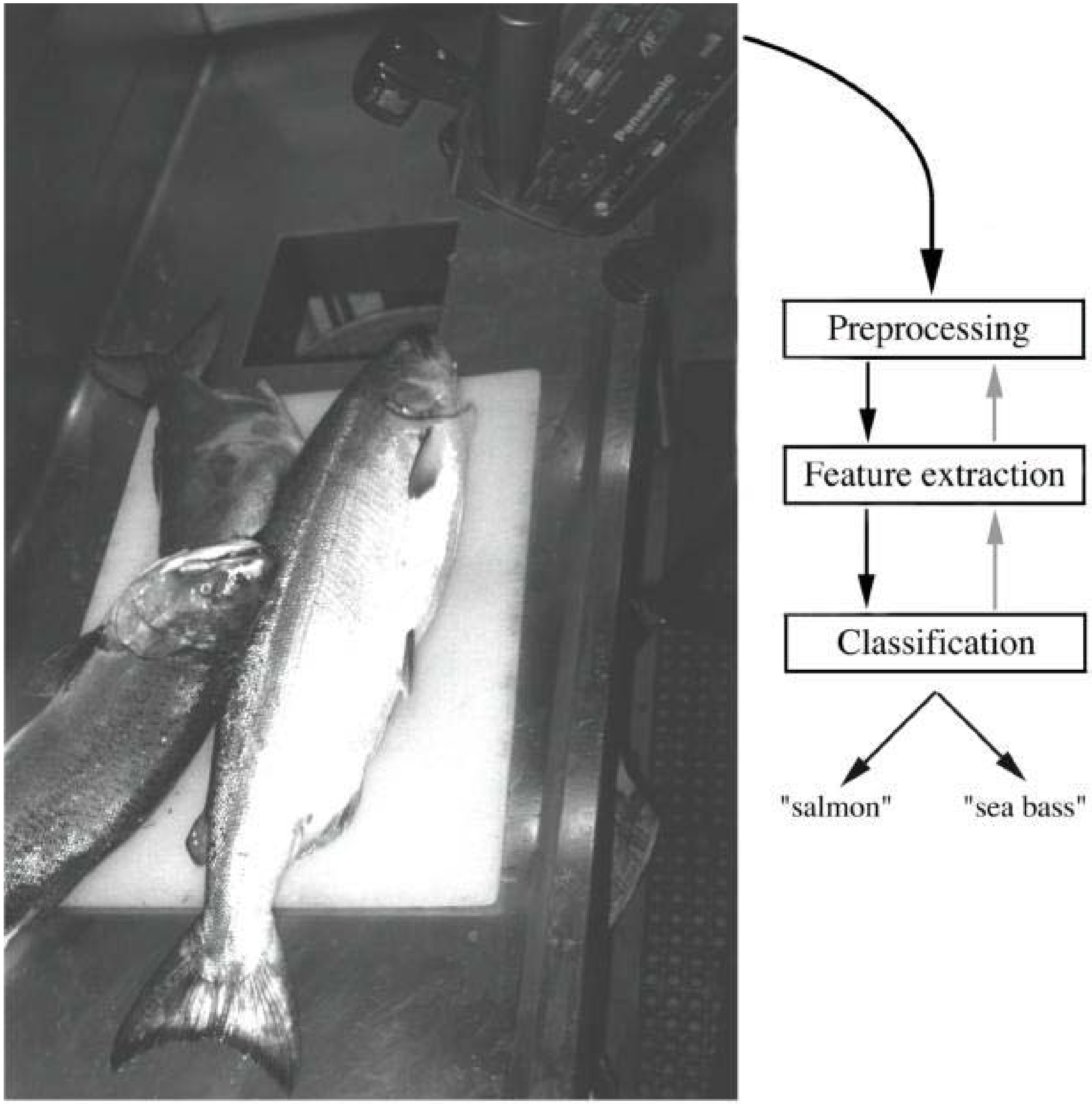
Definitions: feature space $\mathcal{X}$ with $x_i \in \mathcal{X} \subset \mathbb{R}^d$

class labels $y_i \in \{1, \ldots, k\}$

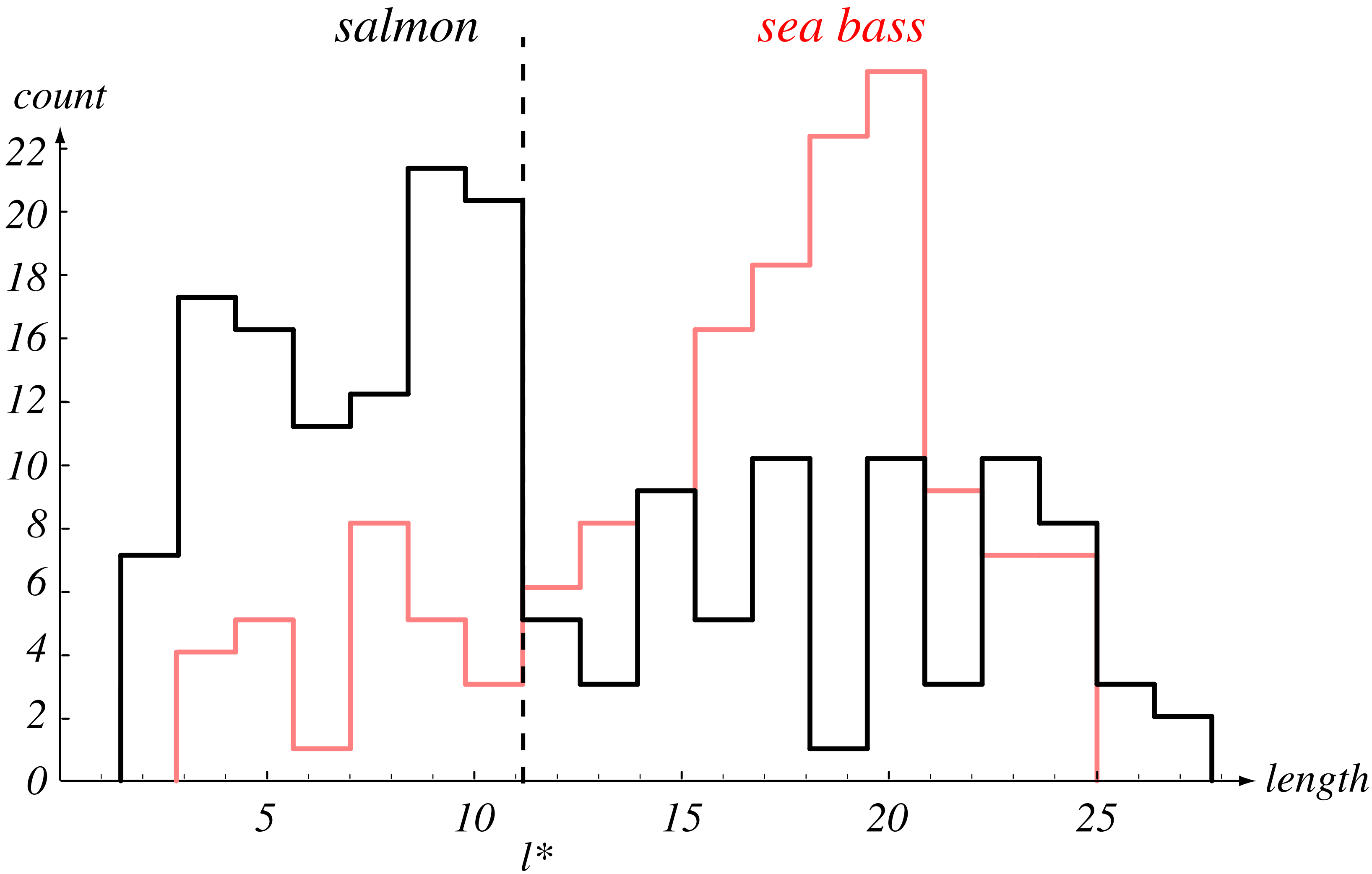
Classifier: mapping $c : \mathcal{X} \to \{1, \ldots, k\}$

k **class problem:** What is $y_{n+1} \in \{1, \ldots, k\}$ for $x_{n+1} \in \mathbb{R}^d$?

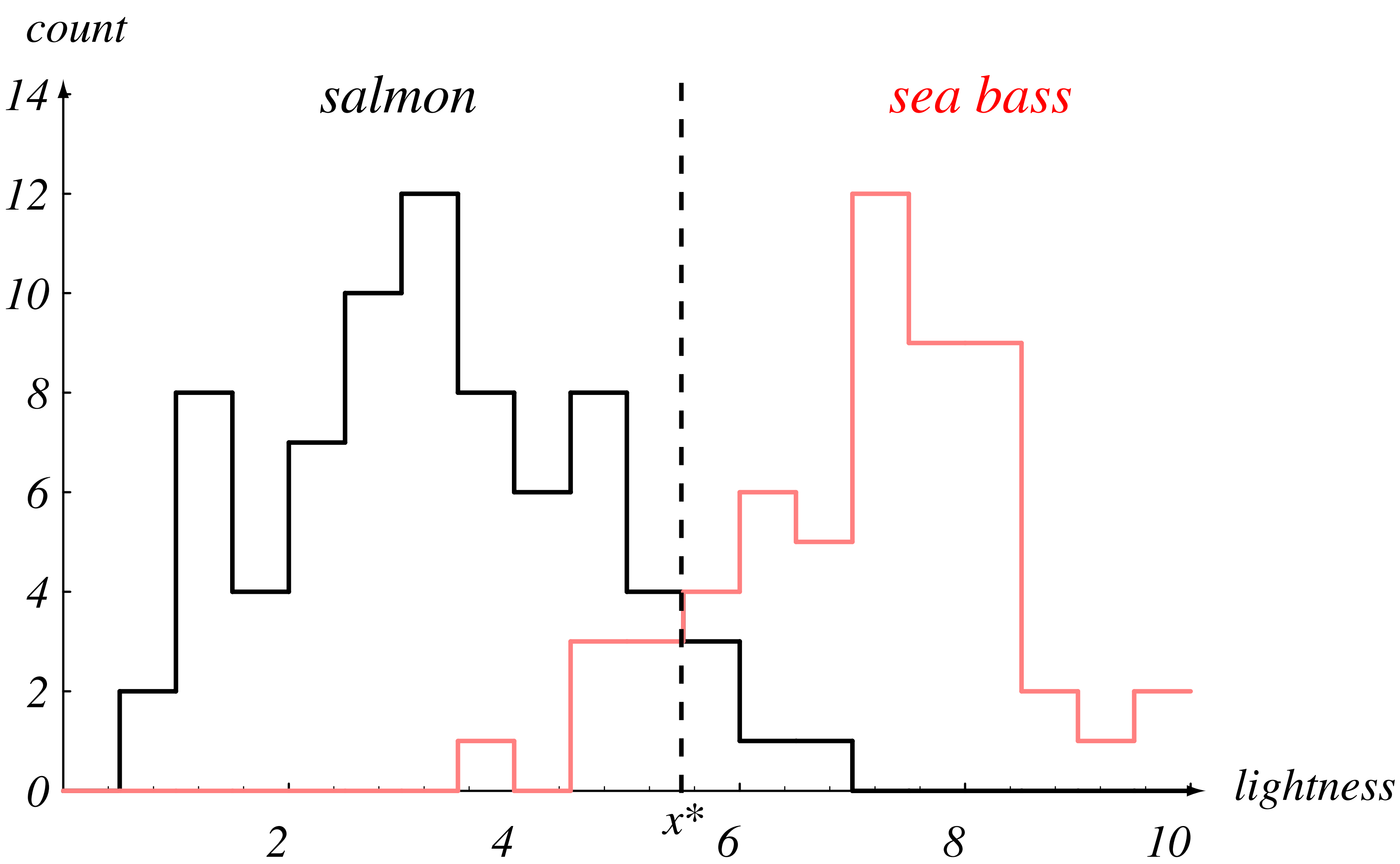
Example of Classification



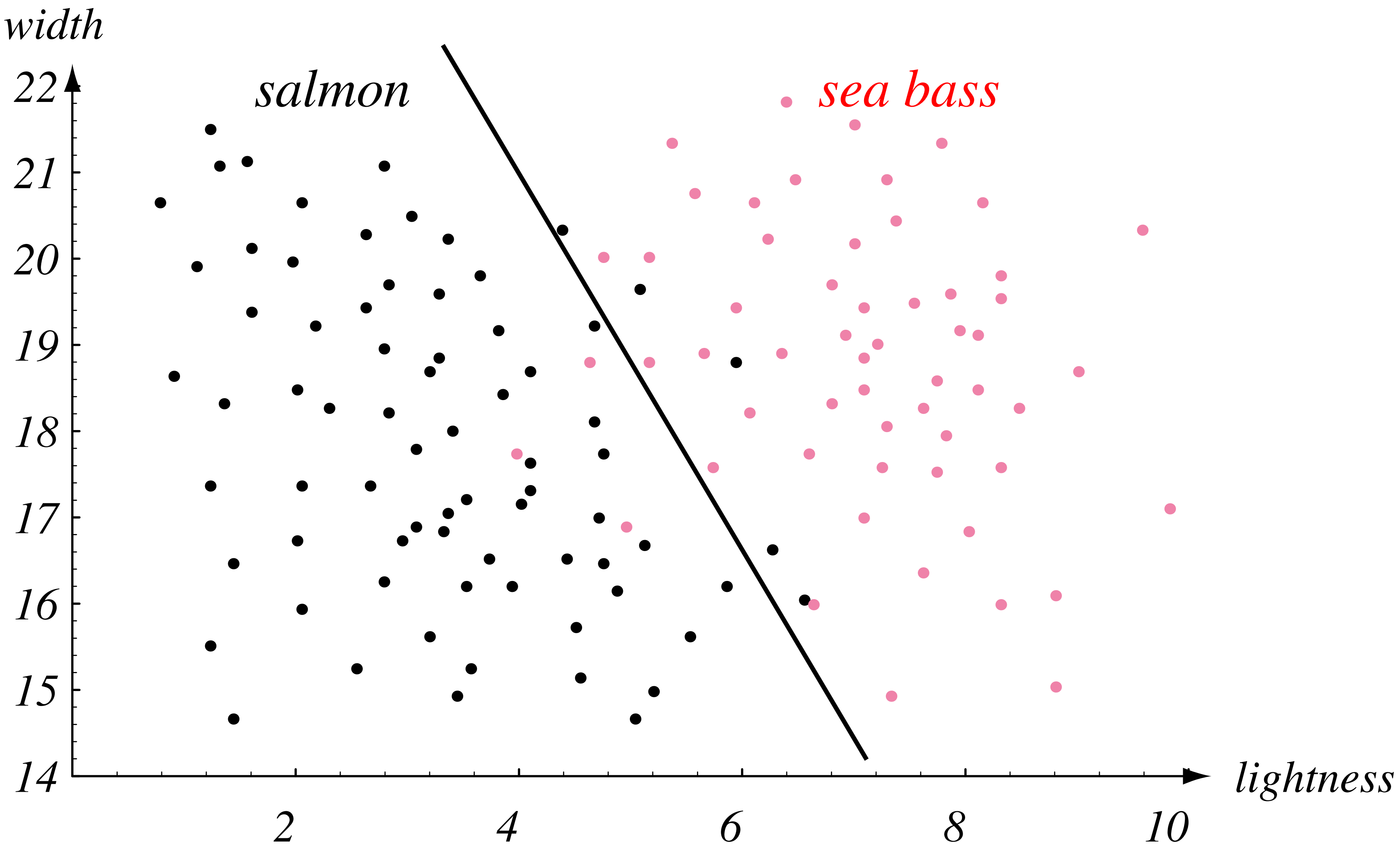
Histograms of Length Values



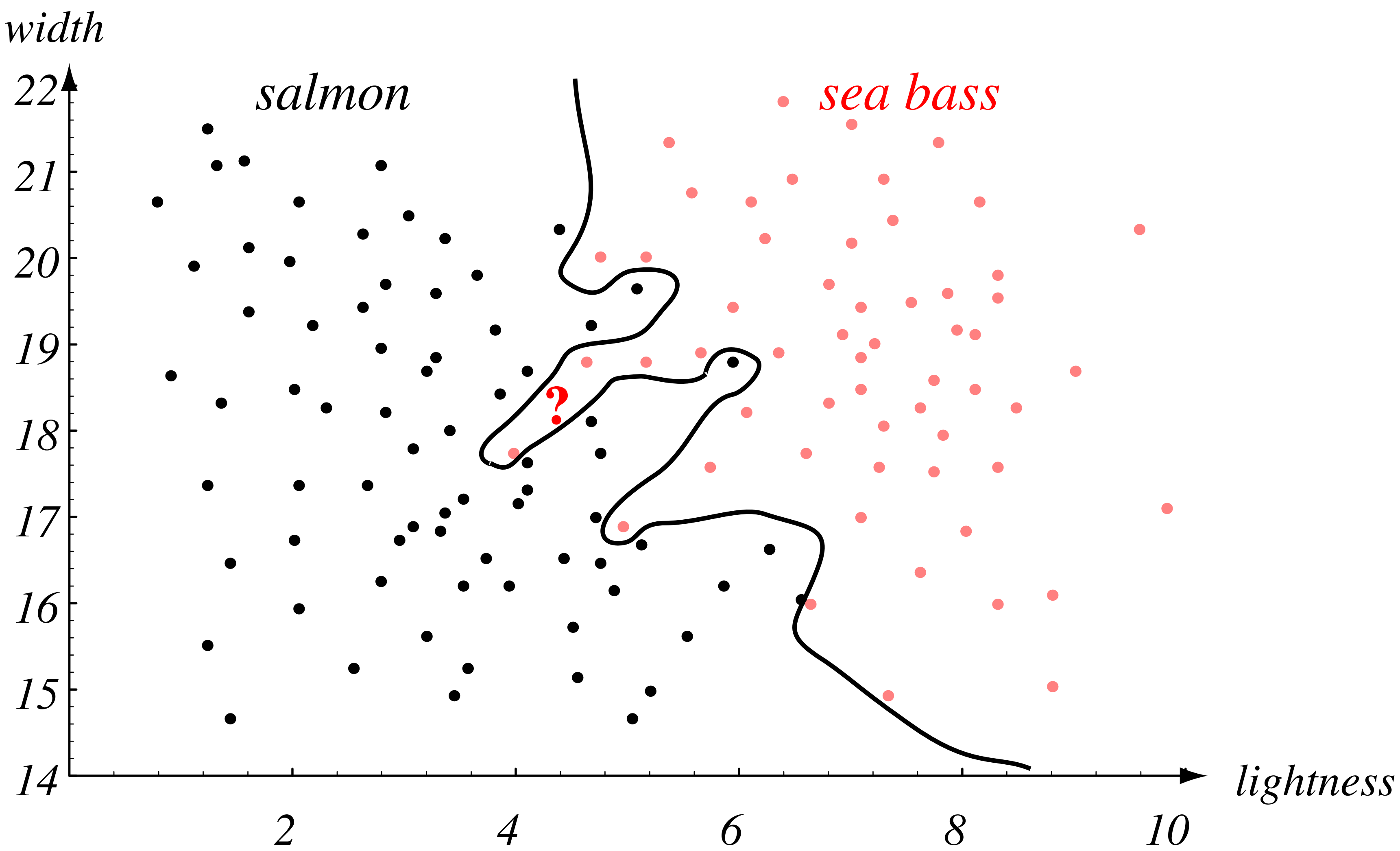
Histograms of Skin Brightness Values



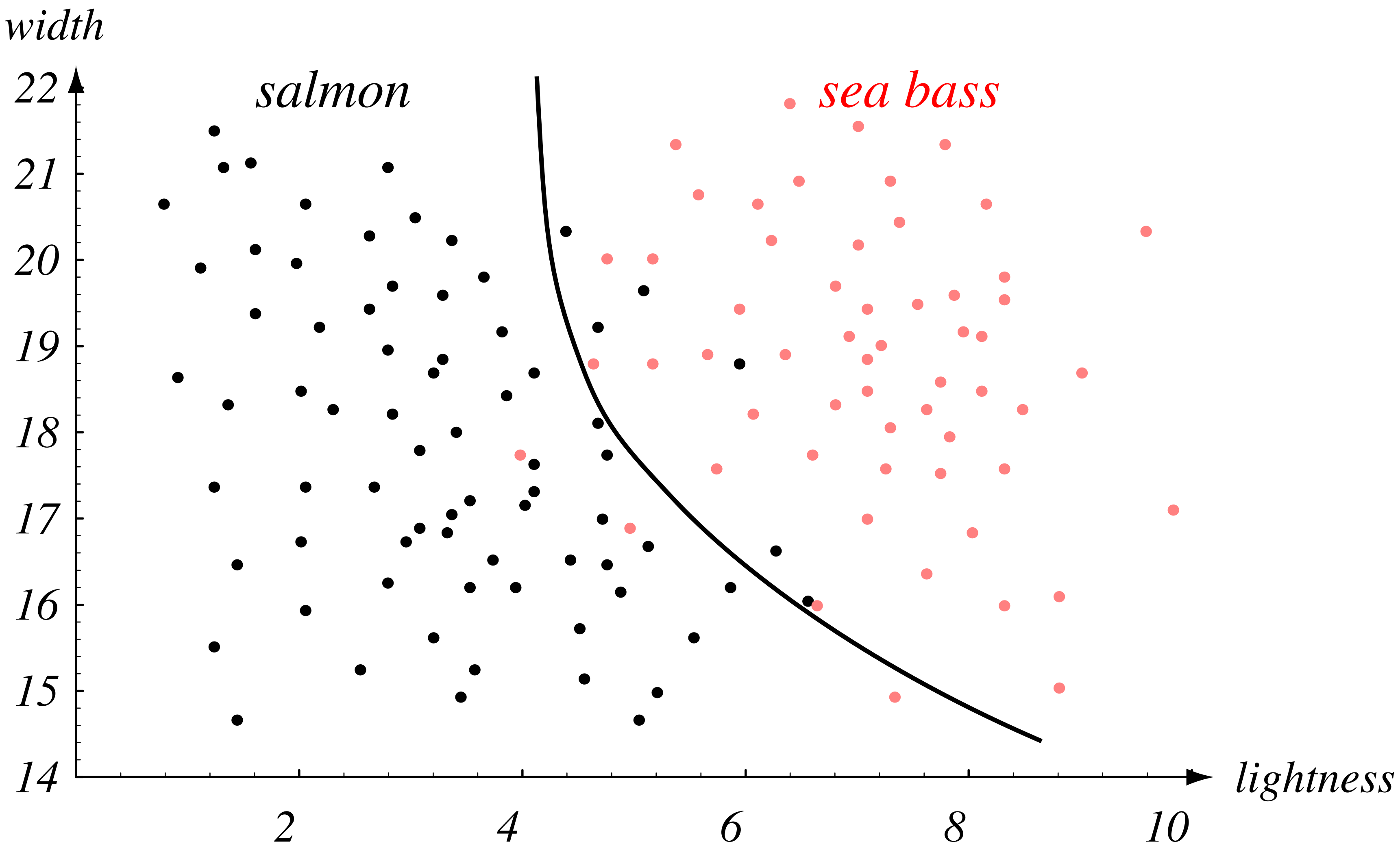
Linear Classification



Overfitting



Optimized Non-Linear Classification

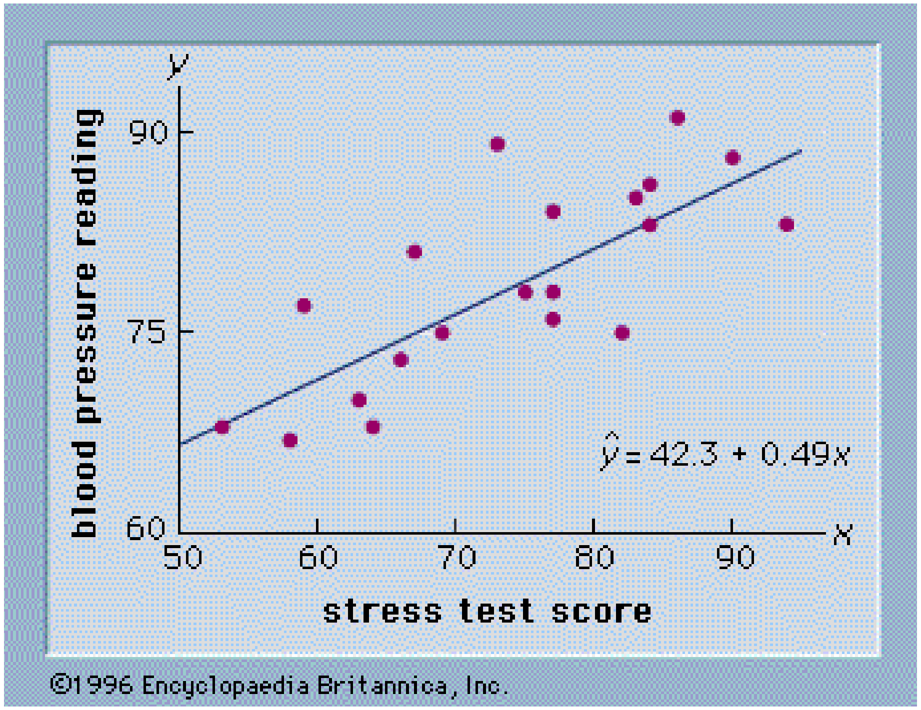


Occam’s razor argument: *Entia non sunt multiplicanda praeter necessitatem!*

Regression

(see Introduction to Machine Learning)

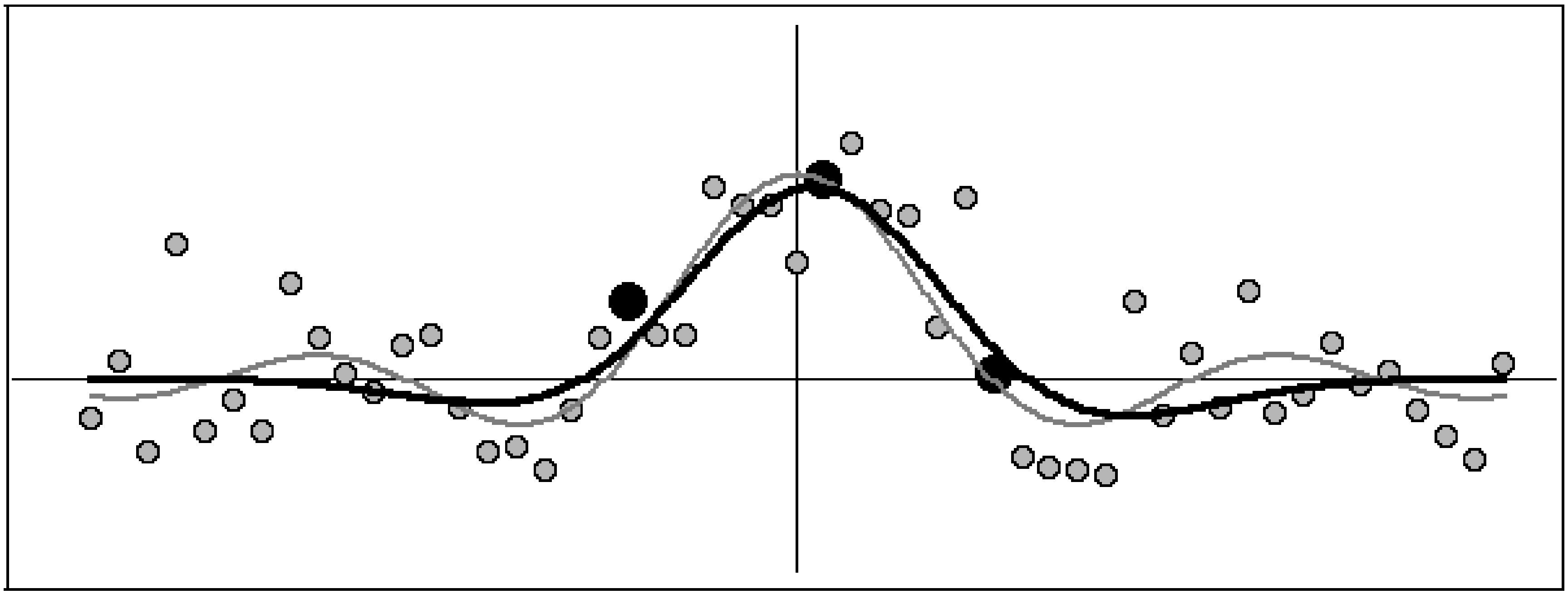
Question: Given a feature (vector) x_i and a corresponding noisy measurement of a function value $(f(x_i) + \text{noise})$ what is the unknown function $f(.) \in$ hypothesis class?



Data: $\mathcal{Z} = \{(x_i, y_i) \in \mathbb{R}^d \times \mathbb{R} : 1 \leq i \leq n\}$

Modeling choice: What is an adequate hypothesis class? Fitting with linear or nonlinear functions?

Nonlinear Regression



50 noisy data from a sinc function sinc$(x) := \sin(x)/x$ (gray) with a regression fit (black).

The Regression Function

Question: What is the optimal estimate of a function $f : \mathbb{R}^d \to \mathbb{R}$ based on noisy data?

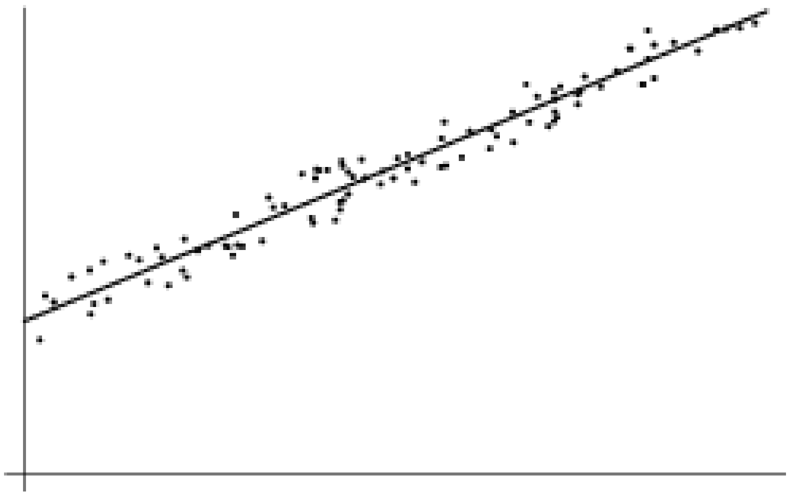
$$y_i = f(x_i) + \eta_i \qquad \leftarrow \qquad \text{noise ?}$$

Solution: the regression function

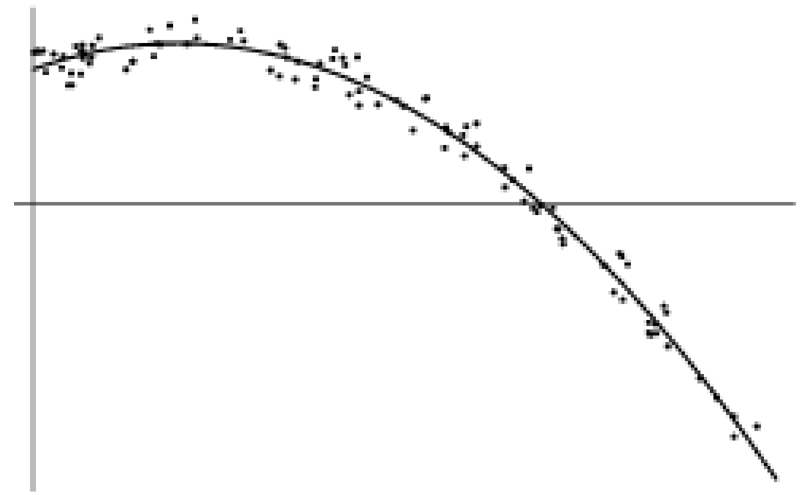
$$y(x) = \mathsf{E}\{y|X = x\} = \int_\Omega y\, p(y|X = x) dy$$

Model selection: **How many data are required to estimate a model or a regression function?**

Examples of linear and nonlinear regression

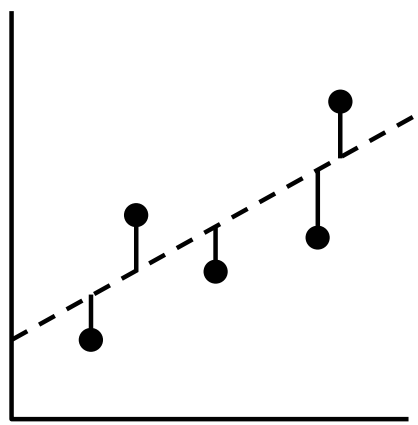


linear regression

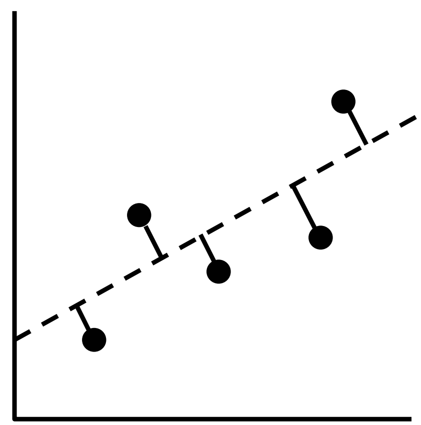


nonlinear regression

How should we measure the deviations?



vertical offsets



perpendicular offsets

Core Questions of Pattern Recognition: Unsupervised Learning

1. **data clustering, vector quantization**

2. hierarchical data analysis; search for tree structures in data

3. visualisation, **dimension reduction**

   (a) *PCA* or principal component analysis
   (b) *ICA*, independent component analysis
   (c) *overcomplete basis*
   (d) *projection pursuit*
   (e) multidimensional scaling, representation of proximity data as Euclidean distances in $\mathbb{R}^d$

Modes of Learning

Reinforcement Learning: weakly supervised learning
Action chains are evaluated at the end.
Backgammon; the neural network *TD-Gammon* gained the world championship! Quite popular in Robotics

Active Learning: Data are selected according to their expected information gain.
Information Filtering

Inductive Learning: the learning algorithm extracts logical rules from the data.
Inductive Logic Programming is a popular sub area of Artificial Intelligence

Taxonomy of Data with some Examples

Data are representations of measurements / observations!

a) monadic data: $\mathcal{O} = \mathcal{O}^{(1)}$

water depth, temperature, pressure, intensity, ...

b) dyadic data: $\mathcal{O} = \mathcal{O}^{(1)} \times \mathcal{O}^{(2)}$

{users} $\times$ {websites}

{gene expression levels} $\times$ {diseases}

{word counts} $\times$ {documents}

pairwise data: $\mathcal{O} = \mathcal{O}^{(1)} \times \mathcal{O}^{(2)}$ with $\mathcal{O}^{(1)} = \mathcal{O}^{(2)}$

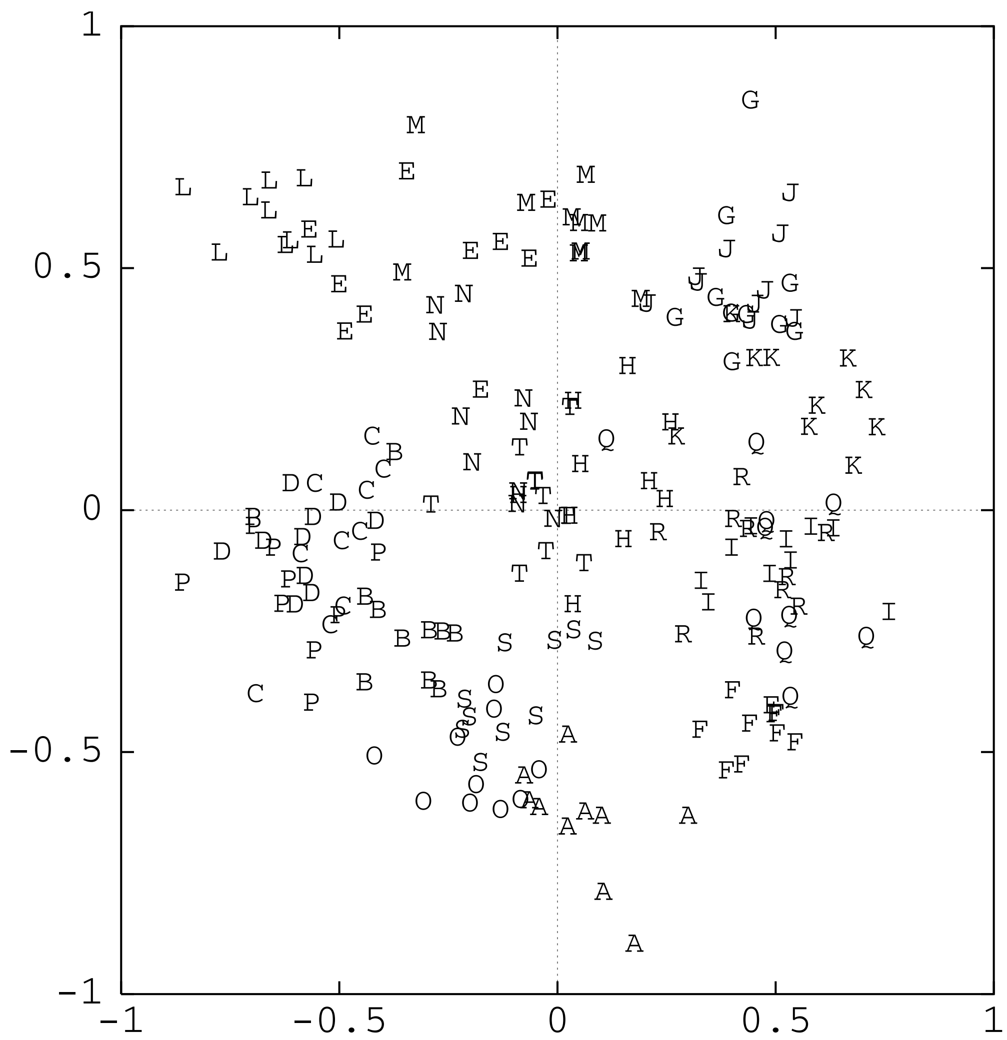
{proteins} $\times$ {proteins}

{image patches} $\times$ {image patches}

c) polyadic data: $R \geq 3$

{test persons} $\times$ {behaviors} $\times$ {traits}

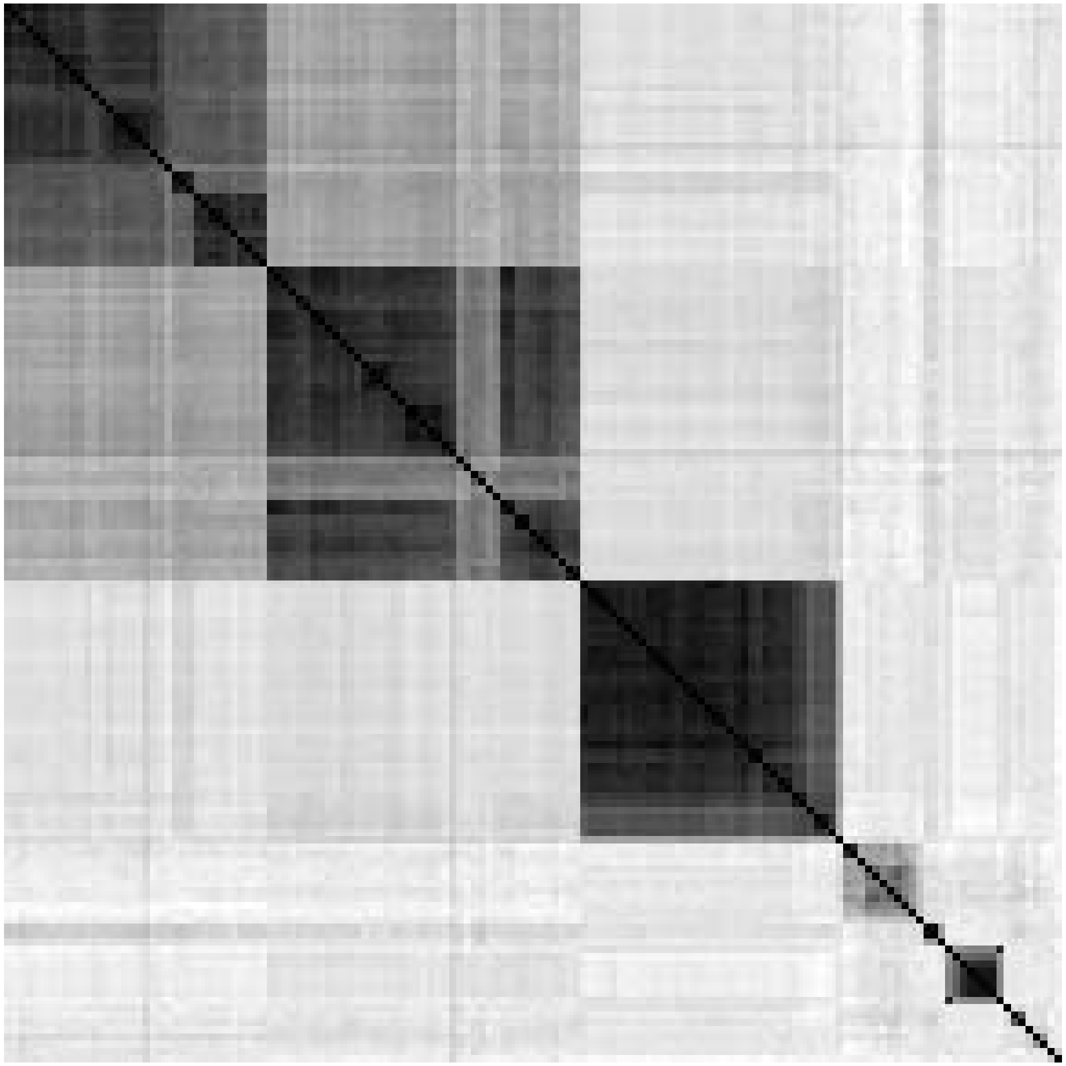
Example for Vectorial Data



Data of 20 Gaussian sources in $\mathbb{R}^{20}$, projected onto two dimensions with Principal Component Analysis.

Example of Relational Data

Pairwise dissimilarity of 145 globins which have been selected from 4 classes of α-globine, β-globine, myoglobins and globins of insects and plants.



Scales for Data

Nominal or **categorial scale**: qualitative, but without quantitative measurements,
e.g. binary scale $\mathcal{F} = \{0, 1\}$ *(presence or absence of properties) or*
taste categories "sweet, sour, salty and bitter.

Ordinal scale : measurement values are meaningful only with respect to other measurements, i.e., the rank order of measurements carries the information, not the numerical differences *(e.g. information on the ranking of different marathon races!?)*

Quantitative scale:

- **interval scale**: the relation of numerical differences carries the information. Invariance w.r.t. translation and scaling (Fahrenheit scale of temperature).
- **ratio scale**: zero value of the scale carries information but not the measurement unit. (Kelvin scale).
- **Absolute scale**: Absolute values are meaningful. (grades of final exams)