Poisson Image Editing

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Seminar Talk by

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Overview

• Guided Image Interpolation
• Discretized Solution
• Editing Operations
• Discussion
Interpolation Problem

- $f^*$: known image values
- $f$: unknown values over region $\Omega$
- Assuming scalar image values
Simple Interpolation

- Maximize smoothness
  \[ \min_f \int_\Omega \| \nabla f \|^2 \]

- Boundary constraints
  \[ f|_{\partial\Omega} = f^*|_{\partial\Omega} \]
Simple Interpolation

- Solution: *Laplace Equation* with Dirichlet boundary conditions

\[ \nabla^2 f = 0, \quad f|_{\partial \Omega} = f^*|_{\partial \Omega} \]

- Membrane solution
- Unsatisfactory due to over-blurring
Guided Interpolation

- \( \mathbf{v} \): guided field
- \( \mathbf{v} \) may be gradient of a function \( g \)
Guided Interpolation

• Minimize difference of gradient fields

\[ \min \int_{\Omega} \| \nabla f - \mathbf{v} \|^2 \]

• Solution: Poisson Equation with Dirichlet boundary conditions

\[ \nabla^2 f = \text{div} \mathbf{v}, \quad f|_{\partial\Omega} = f^*|_{\partial\Omega} \]
Discrete Poisson Solver

- Discretize $\min_{f} \int_{\Omega} \| \nabla f - \mathbf{v} \|^2$ directly by

$$\min_{f|_{\Omega}} \sum_{\langle p,q \rangle \cap \Omega \neq 0} (f_p - f_q - v_{pq})^2$$

$$f_p = f_p^*, \ \forall p \in \partial \Omega$$

for neighbors $p$ and $q$ with

$$v_{pq} = \mathbf{v}(\frac{p+q}{2}) \cdot \overrightarrow{pq}$$

\[\text{Diagram showing neighbors and vector } v_{pq}\]
Discrete Poisson Solver

- Minimum satisfies linear system of equations

If neighborhood $N_p$ overlaps boundary:

$$|N_p|f_p - \sum_{q \in N_p \cap \Omega} f_q = \sum_{q \in N_p \cap \partial \Omega} f_q^* + \sum_{q \in N_p} \nu_{pq}$$

For interior points:

$$|N_p|f_p - \sum_{q \in N_p} f_q = \sum_{q \in N_p} \nu_{pq}$$
Discrete Poisson Solver

- Linear system of equations
  - sparse (banded)
  - symmetric
  - positive-definite

- Irregular shape of boundary requires general solver, such as
  - Gauss-Seidel iteration
  - Multi-grid

- System can be solved at interactive rates
Seamless Cloning

• Importing Gradients from a Source Image $g$

$$\mathbf{v} = \nabla g$$

• Discretize

$$\nu_{pq} := g_p - g_q, \quad \forall \langle p, q \rangle$$
Seamless Cloning Results
Mixing Gradients

• Two Variants
  – \( \mathbf{v} \) averaged from source and destination gradients \( \Rightarrow \) transparency
  – Select stronger one from source and destination gradients:

\[
\mathbf{v}(\mathbf{x}) = \begin{cases} 
\nabla f^*(\mathbf{x}) & \text{if } |\nabla f^*(\mathbf{x})| > |\nabla g(\mathbf{x})| \\
\nabla g(\mathbf{x}) & \text{otherwise}
\end{cases}
\]

Discretization:

\[
u_{pq} = \begin{cases} 
f_p^* - f_q^* & \text{if } |f_p^* - f_q^*| > |g_p - g_q| \\
g_p - g_q & \text{otherwise}
\end{cases}
\]
Mixing Gradients Results

(a) color-based cutout and paste
(b) seamless cloning
(c) seamless cloning and destination averaged
(d) mixed seamless cloning
Mixing Gradients Results

source

destination
Mixing Gradients Results

source/destination  
seamless cloning  
mixed seamless cloning
Texture Flattening

• Preserve only salient gradients

\[ \mathbf{v}(\mathbf{x}) = M(\mathbf{x}) \nabla f^*(\mathbf{x}) \]

with masking function \( M(\mathbf{x}) \) so that

\[ \nu_{pq} = \begin{cases} 
  f_p - f_q & \text{if } \overline{pq} \text{ crosses an edge} \\
  0 & \text{otherwise}
\end{cases} \]
Texture Flattening
Local Illumination Changes

- Approximate tone mapping transformation after Fattal et al. 2002:

\[ \mathbf{v} = \alpha^\beta |\nabla f^*|^{-\beta} \nabla f^* \]

\[ \alpha = 0.2 |\nabla|_{\text{avg}} \]

\[ \beta = 0.2 \]

- Attenuating large gradients
Local Color Changes

• Mix two differently colored version of original image
  – One provides $f^*$ outside
  – One provides $g$ inside
Local Color Changes
Seamless Tiling

• Select original image as $g$
• Boundary condition:
  - $f_{\text{north}}^* = f_{\text{south}}^* = 0.5 (g_{\text{north}} + g_{\text{south}})$
  - Similarly for the east and west
Discussion

Pros

• Very general framework
• No parameter tuning required
• Method does not require precise selection
• Versatile method
  – Seamless cloning, mixing gradients
  – Texture flatening
  – Local changes of illumination and color
  – Seamless tiling
Discussion

Cons

- Cloning requires either of the images to be smooth
- No refined selection is returned
- Minimization only adapts low-frequency content
  - Potential color shift / re-coloring difficult to control
  - Dissatisfactory tiling
  - Cloning requires careful placement of prominent features
Outlook

• More image editing operators
  – Combinations (insert while flattening)
  – Other non-linear operations on gradients
  – More than one source images

• Poisson editing of triangle meshes
  – Feature transfer
  – Detail preserving deformations

• Other editing domains possible?
Thanks
Seamless Tiling
Mixing Gradients Results

source/destination

color transfer

monochrome transfer