Progressive Geometry Compression

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Motivation

- Large (up to billions of vertices), finely detailed, arbitrary topology surfaces
- Difficult manageability of such large amounts of information
  - Computation, Storage, Transmission, Display Strain

Progressive Compression is needed
What Compression Is All About

- Accuracy/bit per vertex
- Definition of a Geometry Error
  - Measure of the geometric distance between 3D objects
- New Problems with Respect to Image Compression
  - No direct correspondence between original and compressed surface

Algorithm

Detailed Mesh

Original Mesh

Semi-regular mesh (MAPS)

Wavelet Transform

Wavelet Coefficients

Geometry Compression

Parameters & Connectivity

2-manifold, arbitrary connectivity

Smooth global parameterization

Semi-regular approximation

Progressive transformation and coding of the approximated shapes

Zerotree Coding

Entropy Coding

Bitrate

Coarse Mesh
For highly detailed, densely sampled meshes, the sample location (vertices + connectivity) do not contribute in improving the geometric distance, and thus the error.

Parameter information is thus contained within the surface, and any rate-distortion improving coding should better take advantage from the geometric information only, normal to the surface.
MAPS algorithm

- “Multiresolution Adaptive Parametrization of Surfaces” based on edge collapsing

Semi-regular approximation is achieved on applying triangle quadrisection to the coarse mesh, and taking advantage of the mapping operated by the algorithm between original and coarse mesh.

Wavelet Transform-1

modeling of a complex object of arbitrary topology at multiple levels of detail

- Replacement of level-N mesh with coarser level-(N-1) mesh + wavelet coefficients
- Generation of “nested” meshes
- Subdivision Rules and Filter-Banks
Wavelet Transform-2

Semi-regular mesh is hierarchically subdivided into a coarser mesh and some details information.

Reconstruction is achieved from the coarse mesh by hierarchical addition of detail information.

Wavelet Transform-3

- **Filter Bank Algorithm**
  - Design of Analysis and Synthesis Filters.
    - Decomposition of a mesh like \( S^{j+1}(x) = \sum \phi_j(\mathbf{x}) + \sum \psi_j(\mathbf{x}) \)

- **Our Paper:**
  - Synthesis: \( p^{j+1} = \begin{bmatrix} P & Q \end{bmatrix} \begin{bmatrix} p^j \\ d^j \end{bmatrix} \)
  - Analysis: solution \([pj, dj]\) of system for a given \(pj+1\)

  - \(P\), low-pass reconstruction filter
  - \(Q\), high-pass reconstruction filter
  - small support
Wavelet Transform-4

Decorrelating effect of the Wv Transform

Vertex position magnitudes for “Venus”

Wavelet Coefficients Magnitudes

Wavelet Coefficients

- Vector valued
- x,y,z components pretty correlated, but
decorrelation in a local frame

Global Frame

Local Frame
Coefficients mostly in the normal direction

- Normal component relevant for geometric information
- Code each component independently
Hierarchical Trees

- Exploit the relationship of wavelet coefficients across bands, and their exponential decay
- Areas with significant information are similar in shape and location
- Non significance in a low frequency band for a particular level of accuracy means with high probability non significance of the children nodes.
- Localizing a zerotree avoids transmitting a large amount of insignificant details with respect to a desired level of accuracy
- Progressive compression, embedded code

Zerotree Coding

Example of the quality of the coding

SPIHT PSNR = 35.12 dB, JPEG PSNR = 31.8 dB (quality factor 15%).

**Zerotree-Howto**

- General: let the decoder see just as much coefficients as needed

- Significance Map: decide which coefficient become significant at a particular bitrate
  - exponentially decreasing threshold

- Refinement Map: decide which coefficients need to be transmitted, as they became significant in a previous pass

- Sign Map: additional information for sign coding

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**Zerotree-Structure**

**Moves:**
- wavelet coefficients are coded as 1-dim array
- lower frequency bands before higher frequency bands
- scan the array and select against a threshold value, which is repeatedly refined
- transmit zerotree information, i.e. as soon as a zero is encountered the corresponding coefficient node is treated as non-relevant.
- as long as a coefficient remains irrelevant a zerotree symbol will be transmitted for it
Further improvement of the bit budget by eliminating redundancy due to non-uniform distribution

- Refinement and sign bits are found to be distributed uniformly
- Significance is a function of bitplane: “early” bitplanes will contain many insignificant coefficients, which will become significant in “later” bitplanes
Results

Rate Distortion Curves:
Improvement by a factor 4!

Relative $L^2$ error ($10^{-4}$)

Bits/vertex

Results

Live Compression...

476 B (ε 40)
1528 B (ε 12)
4163 B (ε 4.7)
26800 B (ε 0.82)
Conclusions

- Very effective compression algorithm
  – smooth appearance, low (hardware) strain
- Many details in very early stages of decompression
- Very fruitful distinction between parameters and geometry
- Still at the beginning of wavelet 3D model compression...

In Fact...

- Effectiveness of compression inherits very much from a bunch of “tested” approaches
- No sound treatment of multiresolution analysis
  – orthogonality, stability...
- “Tentative” choice of wavelets
- Elimination of tangential information (Normal Meshes...)
Demo

Thanks

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• You for your attention….
It’s your turn...

Extra Slides...
Wavelet Transform-2

Ingredients of Multiresolution Analysis (Mallat and Meyer):
Existence of nested linear spaces and of an inner product relative to a subdivision rule.

\[ V^0 \subset V^1 \subset V^2 \subset \cdots \]

\[ W^i := \{ f \in V^{i+1} | \langle f, g \rangle = 0 \quad g \in V^i \} \quad f^{i+1} = f^i + h^i \]

Nested spaces are generated by translations and dilations of a single function, \( \phi(x) \)

\[ \phi(x) = \sum_i p_i \phi(2x - i) \]

\[ V^i := \text{Span}\{ \phi(2^ix - i) | i = -\infty, \ldots, \infty \} \]

Subdivision Rules can be used to define such functions.

Distance Function

- Euclidean Distance (L^2) \( d(X,Y) \) between two surfaces \( X, Y \)

\[ d(X, Y) = \left( \frac{1}{\text{area}(X)} \int_{x \in X} d(x, Y)^2 \, dx \right)^{1/2} \]

- Symmetrize by taking the max of \( d(X,Y) \) and \( d(Y,X) \)